1601. Find the coordinates of the vertex of the second graph, by solving $4 - 2x = 0$.

1602. Put the fraction in its lowest terms before you take the limit.

1603. The point $(\cos\theta, \sin\theta)$ gives the first two. Then the length $|AB|$ is split into $\tan\theta$ and $\cot\theta$, and the lengths $|OA|$ and $|OB|$ are the other two.

1604. (a) Consider the intersection of the lines of action of the first two forces.
(b) Use $\mathbf{F}_{\text{resultant}} = 0$.
(c) Use (a) and (b).

1605. What type of equation is $\text{f}(x) = x$?

1606. The region is a square.

1607. In each part, use the small-angle approximation $\sin x \approx x$, for small x in radians.

1608. (a) Condition on vaccination.
(b) Use the binomial expectation $\mu = np$.
(c) This is a conditional probability. On the tree diagram, restrict the possibility space to those branches representing “coming down with flu”, and find the probability that such a resident is vaccinated.

1609. Just multiply out and simplify the sum.

1610. (a) You need to avoid square rooting negative numbers, and division by zero.
(b) Group the $\sqrt{x}$ terms and take $\sqrt{x}$ out as a factor.
(c) Showing that $y = 1$ is not in the range of f is the same as showing that $x = 1$ is not in the domain of f^{-1}.

1611. Write the sum out longhand. Then you can take out a common factor.

1612. Since the equations are linear, the values of the parameters at the intersection must divide their respective domains in the ratio 1 : 2.

1613. The mean and variance of a binomial distribution are $\mathbb{E}(X) = np$ and $\text{Var}(X) = npq$.

1614. It is not correct.

1615. Use the sine area formula, then the Pythagorean trig identity, then the cosine rule.

1616. Write 3 as $9^{\frac{1}{2}}$, and use index laws.

1617. It’s a semicircle!

1618. (a) Differentiate and substitute $x = 0.8$.
(b) Calculate and consider the value of x_1, with reference to the tangent at $x_0 = 0.8$.

1619. Think of EA as a single “letter”.

1620. These can be modelled on the prototype even and odd cases $y = \sqrt{x}$ and $y = \sqrt[3]{x}$. Such curves are reflections of (parts of) $y = x^2$ and $y = x^3$ in the line $y = x$.

1621. Since the circles all have the same unit radius, their being equidistant is the same as their centres being equidistant.

1622. Since the transformation is a one-dimensional stretch, the area scale factor is the same as the length scale factor.

1623. Translate into algebra, then differentiate using the product rule.

1624. Use t and $t + 2$ as the time variables.

1625. Logarithms are the inverses of exponentials.

1626. (a) Write the area in terms of side length l. Then make l the subject and multiply by 3.
(b) Differentiate part (a) with respect to t.

1627. Substitute the boundary values.

1628. Draw force diagrams for the two objects, setting $F_{\max} = \mu R$ for the limiting case. To work out the direction of friction, consider what would happen if the slope were smooth.

1629. Differentiate with the chain rule. Then evaluate using $\tan \frac{1}{6}\pi = \frac{\sqrt{3}}{3}$ and $\cos \frac{1}{6}\pi = \frac{\sqrt{3}}{2}$.

1630. (a) In a sample size of n, the variance of the mean is scaled by $\frac{1}{n}$.
(b) Calculate the negation, i.e. the probability that the departure of the mean from 100 is less than 5.

1631. Factorise fully. Note that the right-hand factor is a quadratic in x^2.

1632. In this system of coordinates, the weight of the pilot is $-mg\mathbf{k}$.

1633. Find the range of $x \mapsto e^x - 1$ first.

1634. (a) Find the sum of the interior angles of a hexagon, in radians, and set it equal to the sum of a geometric series.
(b) Set up N-R with $f(r) = 128r^6 - 252r + 124$.
(c) Sub a and r into $u_6 = ar^5$.

1635. This is another way of saying "Show that $(2x - 1)$ isn't a factor of $4x^5 - 7x^3 + 2x + 1$."

1636. Variables x and y appear symmetrically as inputs in this equation, so each component of translation should be thought of as a replacement.

1637. Expand $(1+x)^3$ binomially. Then split the fraction up, writing each term as ax^b, before integrating.

1638. Consider the fact that k^2 must be positive.

1639. It is valid.

1640. (a) Find the average acceleration for $t \in [0, 2]$ and for $t \in [2, 4]$. Show that these are not equal.
(b) Write $a = kt$ and integrate with respect to t. Show that there is a value for k and a value for the constant of integration that are consistent with the data.

1641. Solve simultaneously for $\sqrt{x}$ and $\sqrt{y}$ as usual, then deal with the square roots afterwards.

1642. One value comes from the equation not being a quadratic at all; the others need the discriminant.

1643. Solve for intersections, and test the gradient.

1644. (a) Set up and solve an equation in θ.
(b) Use a tree diagram approach. You don't need to draw the tree diagram, however. You aren't using part (a) in part (b).

1645. This is a quadratic in e^{2x}.

1646. Explain why irrationals must be excluded, and why zero must be excluded.

1647. This can be done simply by symmetry, or explicitly using *suvat*.

1648. Test the discriminants.

1649. (a) Solve simultaneously for intersections, and test the gradients.
(b) Use Pythagoras.

1650. The equations are the same.

1651. "Concurrent" means that all three circles meet at a single point. Find the intersections of the first two circles and test them in the third.

1652. (a) Use the chain rule.
(b) The best linear approximation to a curve is also known as the tangent line.

1653. (a) There are two successes: LL and RR.
(b) There are three successes: LR, RL and NN.

1654. Simplify using the definition of a logarithm and, for the second expression, an index law.

1655. (a) The mean transforms linearly.
(b) Express the mean of x_i^2 algebraically using sigma notation. Rearrange the usual variance formula to make this expression the subject.

1656. (a) Find the horizontal acceleration using NII, then use *suvat* with a time of 8 seconds.
(b) Find the horizontal speed at landing, and use Pythagoras.

1657. Find the gradient of the first line, and then use its negative reciprocal.

1658. (a) Divide top and bottom by e^x.
(b) Multiply top and bottom by e^x.

1659. This is a quadratic in 7^x.

1660. Consider three consecutive terms u_n, u_{n+1}, u_{n+2} in a progression. Find u_{n+1} in terms of the other two if the sequence is
① arithmetic, by equating differences,
② geometric, by equating ratios.

1661. Rearrange to make b the subject of the second equation. Substitute this into the first equation and rearrange.

1662. Draw a six by six probability space, and restrict it. This is a regular conditional probability, albeit couched in fancy language.

1663. Differentiate implicitly, using the chain rule. This means treating y as a function of x. If this needs clarification, let $y = f(x)$.

1664. (a) Assumptions are required to say ① that the block on the table accelerates at a, and ② that the tension is the same throughout the string.
(b) The string is taut, so you can resolve along it for the whole system.

(c) The string exerts two tensions on the pulley: T acting along each of the directions of the string. So, the pulley exerts the NIII pairs of these forces. They combine by Pythagoras.

1665. Consider the periodicity of the sine function.

1666. The first statement is false, the second true.

1667. Consider the cases $x < 0$, $x = 0$ and $x > 0$. Where it is defined, the gradient of the graph is 0.

1668. You can't use the real factor theorem here. So, instead, attempt the factorisation explicitly with $(x^2 + 4)(ax^2 + bx + c)$.

1669. (a) Consider the fact that the first two terms have even degree. Alternatively, complete the square, noting that the resulting squared term cannot be rendered zero. Its minimum occurs when $x = 0$.
(b) Reciprocating the range $[k, \infty)$ gives $(0, 1/k]$.

1670. (a) There is one successful outcome.
(b) There are six successful outcomes.

1671. Translate "$f'(x) - g'(x)$ is linear in x" into algebra, and integrate.

1672. (a) i. Use the usual projectile method: find the time of flight with a vertical *suvat*, then the horizontal range with that time of flight.
ii. Do likewise. The horizontal equation will now require a *suvat*.
(b) Consider the fact that the range in ii. is larger than the range in i.

1673. (a) A function is only invertible if it is one-to-one. Hence, $(-\infty, 2]$ or $[2, \infty)$ must give the two halves of the parabola.
(b) Consider $f(x)$ in completed square form.

1674. (a) Take out a factor of x^2. Then take a graphical approach to show that $x^5 - 1$ has exactly one root, which is a single root.
(b) All you need is in part (a).
(c) You are looking for x values for which the curve is above the x axis.

1675. A point of inflection requires that the second derivative be zero. The words *order* and *degree* are interchangeable regarding polynomials: they refer to the largest power.

1676. In each case, replace input variables. Reflection in $y = x$ is a switch $x \leftrightarrow y$; reflection in $x = 0$ is a replacement of x with $-x$.

1677. Split the shaded area into symmetrical segments, and calculate the area of one as the area of a sector minus the area of a triangle.

1678. The best linear approximation to $f(x)$ at $x = a$ is $g(x)$ such that $y = g(x)$ is the tangent line to $y = f(x)$ at $x = a$. So, solve to find the roots. Then set up $y = x^5 - x$, and find the equations of the tangents at the x intercepts.

1679. Divide top and bottom by x^2 before taking the limit.

1680. The parabola must be negative, and its vertex must lie on the y axis.

1681. Use the binomial expansion.

1682. This is not true. Equilibrium isn't necessary.

1683. Consider the fact that these curves are reflections in $y = x$. Then use the discriminant.

1684. Find an ordinal formula $u_n = an^2 + bn + c$, using the fact that the second difference is $2a$. Then set up an equation $u_{n+1} - u_n = 60$ and solve.

1685. Multiply up, gather terms in x^3, and factorise.

1686. A Venn diagram might help.

1687. Sketch the regions on a number line, thinking of the inequalities in terms of *distance* from a point.

1688. Use the factor theorem to find A. Then take out a factor of $2x-1$. Do this directly, or use polynomial long division if needs be.

1689. Show that the denominator is never zero.

1690. You only need to find two perpendicular bisectors. From these you can find the centre, and hence test distances with Pythagoras.

1691. (a) The three lines of action must be concurrent.
(b) Resolve perpendicular to the line of action of the third force.

1692. This is the reverse chain rule. Since the inside function is linear, of the form $ax + b$, a factor of $\frac{1}{a}$ is required.

1693. Since the two lines do not intersect, they must be parallel.

1694. You can consider any of: factorisation, completing the square, or differentiation to find the vertex.

1695. Consider, in any given suit, the number of sets of the form $\{8, 9, 10, \text{J}, \text{Q}\}$. You'll need the factorial formula for ${}^n\text{C}_r$.

1696. Split the exponential, and then write e as 10^k.

1697. Think carefully about which values change across the different terms of the sum and which don't, i.e. which letters represents constants and which represent variables.

1698. (a) i. This is about the string.
   ii. This is mainly about the pulleys, but also about the string.
   (b) You need one acceleration a, but two different tensions T_1 and T_2.
   (c) Add the three equations of motion. This will cancel the internal NIII pairs, which are the tensions. This is equivalent to resolving along the (taut) strings for the entire system.

1699. A vertical line such as $x = \frac{\pi}{2}$ must intersect any graph $y = \text{f}(x)$ if the value $x = \frac{\pi}{2}$ is in the domain of the function f.

1700. Find the perpendicular bisector.

——— END OF 17TH HUNDRED ———